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THEORY OF PLASTIC DEFORMATION FOR MULTICOMPONENT POROUS MEDIA

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The creation of dispersed-reinforced materials having specific technical properties is achieved by the bonding of heterogeneous metals through plastic deformation of powdered mixtures. The properties of the composites formed in this way are qualitatively distinguished from those of the component materials. To a significant degree this is due to the presence of pores. Theoretical models of the plastic deformation of porous media can be used in the choice of methods and regimes of pressure moulding employed to obtain quality manufactured products. In this work we investigate the features of plastic deformation of porous media containing dispersed inclusions. A method is employed which gives an approximate expression for the composite dissipation function [1-8]. We obtain the conditions under which the inclusions behave as rigid particles or deform together with the matrix.

1. We examine a rigid-plastic material made up of a connecting matrix with a uniform distribution of inclusions and pores in it. The matrix and inclusions satisfy the von Mises condition with plastic flow limits k_0 and k_1, respectively. The problem consists of constructing approximate expressions for the composite dissipation function $D^*(\langle\varepsilon_{ij}\rangle)$, which in combination with an associated stress rule $\langle\sigma_{ij}\rangle = \partial D^*/\partial\langle\varepsilon_{ij}\rangle$ determines the plasticity conditions [1-8]. Here σ_{ij}, ε_{ij} are components of the stress and plastic strain rate tensors, and the angular brackets denote averaging of the field over the material volume.

The dissipation function of the macroscopic medium $D^*(\langle\varepsilon_{ij}\rangle)$ is obtained as the minimum value of the dissipation rate in a unit of macroscopic volume V of the porous body:

$$D = \frac{1}{V}\int\limits_{V_0} k_0 \sqrt{\varepsilon_{ij}\varepsilon_{ij}}\,dV + \frac{1}{V}\int\limits_{V_1} k_1 \sqrt{\varepsilon_{ij}\varepsilon_{ij}}\,dV \tag{1.1}$$

($V = V_S + V_2$, where the solid-phase volume is $V_S = V_0 + V_1$; V_0, V_1, and V_2 are the volumes of the matrix, inclusions, and pores, respectively).

By representing the integral over V_0 in the form of the difference between the integrals over V_S and V_1, we have the functional

$$D = k_0\langle\sqrt{\varepsilon_{ij}\varepsilon_{ij}}\rangle_S - (k_0 - k_1)\,\langle\sqrt{\varepsilon_{ij}\varepsilon_{ij}}\rangle_1, \tag{1.2}$$

which for $k_1 = k_0$ reduces to the expression for the dissipation function of a porous body with a homogeneous solid phase [2]. The indices after the angular brackets in (1.2) signify averaging over the appropriate phase.

Following [2-7], we employ the approximate relations

$$\langle\sqrt{\varepsilon_{ij}\varepsilon_{ij}}\rangle_S \approx \sqrt{\langle\varepsilon_{ij}\varepsilon_{ij}\rangle_S},\quad \langle\varepsilon_{ij}\varepsilon_{ij}\rangle_n \approx \langle\varepsilon_{ij}\rangle_n\langle\varepsilon_{ij}\rangle_n, \tag{1.3}$$

where n = 1, 2. Using $2\varepsilon_{ij} = v_{i,j} + v_{j,i}$, the value of $\langle\varepsilon_{ij}\rangle_2$ is determined by the displacement rate v_i at the pore surface according to the Gauss–Ostrogradskii formula. In

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[3, 4] a qualitative estimate of the first approximation in (1.3) was made. The second approximation most closely corresponds to spherically shaped inclusions and pores [4, 5].

Using a prime to denote the fluctuation of the field with respect to its average value, we write

$$\varepsilon_{ij} = \langle\varepsilon_{ij}\rangle + \varepsilon'_{ij}, \quad \langle\varepsilon_{ij}\varepsilon_{ij}\rangle = \langle\varepsilon_{ij}\rangle\langle\varepsilon_{ij}\rangle + \langle\varepsilon'_{ij}\varepsilon'_{ij}\rangle. \tag{1.4}$$

We introduce notation for the invariants

$$I_0 = \sqrt{\langle\varepsilon_{ij}\rangle\langle\varepsilon_{ij}\rangle}, \ I_n = \sqrt{\langle\varepsilon_{ij}\rangle_n\langle\varepsilon_{ij}\rangle_n} \ (n = 1, 2),$$
$$I = \left(I_0^2 + \langle\varepsilon'_{ij}\varepsilon'_{ij}\rangle - c_2 I_2^2\right)^{1/2}. \tag{1.5}$$

By using (1.2)-(1.5), expression (1.1) can be transformed to

$$D = k_0\sqrt{1 - c_2}\, I - \hat{k} c_1 I_1 \tag{1.6}$$

($c_n = V_n/V$ is the volume concentration of phase n, $\hat{k} = k_0 - k_1$).

Averaging over V_n reduces to averaging over the entire volume V if the integrand is multiplied by the function $\varkappa_n$, which takes the value 1 inside V_n and is equal to 0 at other points in the material. For such a function

$$\langle\varkappa_n\rangle = c_n, \quad \langle\varkappa'_1\varkappa'_2\rangle = -c_1c_2, \quad \langle\varkappa'_n\varepsilon'_{ij}\rangle = c_n(\langle\varepsilon_{ij}\rangle_n - \langle\varepsilon_{ij}\rangle). \tag{1.7}$$

From the assumption of homogeneity of the deformed state of the pores (1.3) and the solid phase incompressibility condition for the porous body, we have the relation $\varepsilon_{kk} = \langle\varepsilon_{ii}\rangle_2\varkappa_2$, whose average over the macroscopic volume V gives

$$\langle\varepsilon_{kk}\rangle_2 = \varepsilon_0/c_2, \ \varepsilon_0 = \langle\varepsilon_{kk}\rangle. \tag{1.8}$$

The condition that the functional (1.6) be a minimum for fluctuations v'_i leads to

$$\frac{k_0\sqrt{1-c_2}}{I}\left(\varepsilon'_{ij,j} - \langle\varepsilon_{ij}\rangle_2\varkappa'_{2,j}\right) - \frac{\hat{k}}{I_1}\langle\varepsilon_{ij}\rangle_1\varkappa'_{1,j} + p'_{,i} = 0, \tag{1.9}$$

which is supplemented by the incompressibility condition for the solid phase and the Cauchy relations written for fluctuations:

$$v'_{k,k} = \langle\varepsilon_{ii}\rangle_2\varkappa'_2, \quad 2\varepsilon'_{ij} = v'_{i,j} + v'_{j,i} \tag{1.10}$$

[p' is the Lagrangian multiplier for incompressibility condition (1.10)].

The solution to system (1.9), (1.10) in a spectrally dense space with transformation parameters ξ_i has the form

$$\varepsilon_{ij}(\xi) = \left[\mu\langle\tilde{\varepsilon}_{kl}\rangle_1\varkappa'_1(\xi) + \langle\tilde{\varepsilon}_{kl}\rangle_2\varkappa'_2(\xi)\right]\left(\frac{\xi_k\xi_j}{\xi^2}\delta_{li} + \frac{\xi_l\xi_i}{\xi^2}\delta_{kj} - 2\frac{\xi_i\xi_j\xi_k\xi_l}{\xi^4}\right) + \varepsilon'_{kk}(\xi)\frac{\xi_i\xi_j}{\xi^2}. \tag{1.11}$$

Here

$$\mu = \frac{\hat{k}}{k_0\sqrt{1-c_2}}\,\frac{I}{I_1}; \tag{1.12}$$

$\xi^2 = \xi_i\xi_i$; the functions differ in their form as indicated by argument. The ~ sign denotes the deviatoric portions of the tensors.

Solution (1.11) allows us to express $\langle\varkappa'_n\varepsilon'_{ij}\rangle$ in terms of $\langle\varepsilon_{ij}\rangle_n$, (n = 1, 2) [1-5]. Substitution of the appropriate expressions into (1.7) leads to:

$$3\langle\tilde{\varepsilon}_{ij}\rangle_2/5 = (1 - 2\mu/5)\langle\tilde{\varepsilon}_{ij}\rangle_1; \tag{1.13}$$

$$[(1 + 2c_2/3) - \mu\gamma]\langle\tilde{\varepsilon}_{ij}\rangle_1 = \langle\tilde{\varepsilon}_{ij}\rangle, \ \gamma = 2(3 + 2c_2 - 3c_1)/15. \tag{1.14}$$

With the help of (1.7) and (1.9) one obtains [4]

$$\langle\tilde{\varepsilon}'_{ij}\tilde{\varepsilon}'_{ij}\rangle = c_2\langle\tilde{\varepsilon}_{ij}\rangle_2(\langle\tilde{\varepsilon}_{ij}\rangle_2 - \langle\tilde{\varepsilon}_{ij}\rangle) + \mu c_1\langle\tilde{\varepsilon}_{ij}\rangle_1(\langle\tilde{\varepsilon}_{ij}\rangle_1 - \langle\tilde{\varepsilon}_{ij}\rangle) +$$
$$+ \frac{2}{3}\langle\varepsilon_{kk}\rangle_2 c_2(1 - c_2),$$

whose use in (1.5) after a transformation using (1.13) and (1.14) gives

$$I^2 = \frac{3(1-c_2)}{3+2c_2} I_0^2 + \frac{2(1-c_2)}{3c_2} \varepsilon_0^2 + \frac{3c_1\gamma}{3+2c_2} I_\varepsilon^2,$$
$$(\gamma I_\varepsilon)^2 = \left(\frac{3+2c_2}{3}\langle\tilde{\varepsilon}_{ij}\rangle_1 - \langle\tilde{\varepsilon}_{ij}\rangle\right)\left(\frac{3+2c_2}{3}\langle\tilde{\varepsilon}_{ij}\rangle_1 - \langle\tilde{\varepsilon}_{ij}\rangle\right). \tag{1.15}$$

One of the features of porous materials undergoing plastic deformation is their strengthening during compaction and softening when decompacted. An increase in the stress level in the porous bonding can lead to deformation of the inclusions which are initially rigid. The reverse process is also possible, when the material is decompacted and the plastic inclusions begin to behave as rigid particles. These features of composite deformation for fixed loads $\langle\sigma_{ij}\rangle$ occur depending on the properties and concentrations of the component materials.

The volume concentration of inclusions $c_1 = V_1/V$ is a function of the porosity c_2; therefore below we shall switch to the concentration of the inclusions in the solid phase of the material $c_1{}^* = V_1/T_S$ = const. It is not difficult to show that $c_1 = c_1{}^*(1 - c_2)$. If the inclusions behave as rigid particles ($\langle\varepsilon_{ij}\rangle_1 = 0$, $I_\varepsilon{}^2 = (I_0/\gamma)^2$), then I is found from (1.15) as a function of the variables I_0, ε_0, and according to (1.6) we can define a dissipation function for a porous body with rigid inclusions

$$D_1^* = k_0\sqrt{\beta(1 + c_1^*/\gamma) I_0^2 + \beta\varepsilon_0^2/9\alpha}$$
$$(\alpha = c_2/(6 + 4c_2),\ \beta = 3(1 - c_2)^2/(3 + 2c_2)). \tag{1.16}$$

In the process of inclusion deformation $\langle\varepsilon_{ij}\rangle_1 \neq 0$. By using (1.12), the set of equations (1.14) leads to

$$I_1 = \frac{3}{3+2c_2}\left(I_0 + \frac{\widehat{k}\gamma I}{k_0\sqrt{1-c_2}}\right),\quad I_\varepsilon^2 = (\widehat{k} I / k_0\sqrt{1-c_2}), \tag{1.17}$$

Substitution of these into (1.15) and (1.16) gives

$$k^*\sqrt{1-c_2}\, I = k_0\sqrt{\beta I_0^2 + \beta\varepsilon_0^2/9\alpha}; \tag{1.18}$$
$$D_2^* = k^*\sqrt{\beta I_0^2 + \beta\varepsilon_0^2/9\alpha} - \widehat{k}\beta^* I_0; \tag{1.19}$$
$$k^* = \sqrt{k_0^2 - 3c_1^*\gamma\widehat{k}^2/(3 + 2c_2)};\quad \beta^* = \frac{3c_1^*(1-c_2)}{3+2c_2} \tag{1.20}$$

($D_2{}^*$ is the dissipation function for a composite with deformable inclusions).

The dissipation function (1.16) and the associated stress rule $\langle\sigma_{ij}\rangle = \partial D_1{}^*/\partial\langle\varepsilon_{ij}\rangle$ determine the conditions of plastic deformation of a porous body with rigid incluions

$$J_0^2 + \alpha(1 + c_1^*/\gamma)\sigma_0^2 = k_0^2\beta(1 + c_1^*/\gamma)$$
$$\left(J_0^2 = \langle\tilde{\sigma}_{ij}\rangle\langle\tilde{\sigma}_{ij}\rangle,\quad \sigma_0 = \langle\sigma_{kk}\rangle\right). \tag{1.21}$$

The associated flow rule for plasticity conditions (1.21) is written in the form

$$\langle\tilde{\varepsilon}_{ij}\rangle = \lambda_1\langle\tilde{\sigma}_{ij}\rangle,\quad \varepsilon_0 = \lambda_1 3\alpha(1 + c_1^*/\gamma)\sigma_0. \tag{1.22}$$

Similarly, use of the dissipation function (1.19) allows us to obtain the plasticity condition

$$(|J_0| + \widehat{k}\beta^*)^2 + \alpha\sigma_0^2 = k^{*2}\beta \tag{1.23}$$

and the associated flow rule for a porous body with deformable inclusions

$$\langle\tilde{\varepsilon}_{ij}\rangle = \lambda_2(|J_0| + \widehat{k}\beta^*)\langle\tilde{\sigma}_{ij}\rangle/|J_0|,\ \varepsilon_0 = \lambda_2 3\alpha\sigma_0$$
$$\left(|J_0| = \sqrt{J_0^2}\right). \tag{1.24}$$

We have given the conditions for simultaneous deformation of matrix and inclusions and also those when the inclusions are rigid. In the first case, composite deformation is described by relations (1.23) and (1.24); the second case by (1.21) and (1.22).

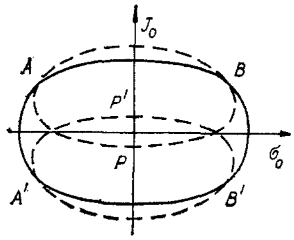

Fig. 1

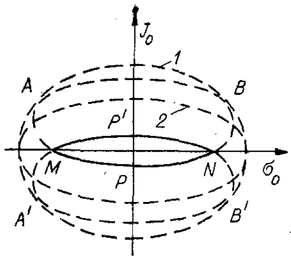

Fig. 2

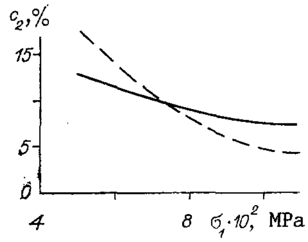

Fig. 3

2. Let the plastic limit of the inclusions k_1 be greater than that of the matrix k_0: then $\hat{k} < 0$. The condition of deformation of the inclusions $I_1 > 0$ and (1.17) require fulfillment of the inequality $k_0\sqrt{1 - c_2}\cdot I_0 + \hat{k}\gamma I > 0$, whose solution, using (1.18) leads to

$$\gamma\varepsilon_0 < \sqrt{3c_2\gamma^*/2}\, I_0, \quad \gamma^* = \frac{k_0^2}{\hat{k}^2} - \frac{(2 + 3c_1^*)\,\gamma}{5}. \tag{2.1}$$

If $\gamma^* > 0$, the inclusions either deform together with the matrix or else remain rigid, depending on whether or not (2.1) is met. It can be shown that $\gamma^* > 0$ ensures that the expression under the radical in (1.20) is also positive.

Substituting I_0 and ε_0 obtained from (1.24) into (2.1), after a transformation using (1.23) and expressions for the functions α, β, γ, and γ^*, we obtain

$$|J_0| > a, \quad |\sigma_0| < b, \quad a = -\hat{k}(1 - c_2)(2 + 3c_1^*)/15, \quad b = -\hat{k}\sqrt{\gamma^*\beta/\alpha}. \tag{2.2}$$

Conditions (2.2) in the σ_0, J_0 plane determine the region where composite deformation is described by (1.23) and (1.24). In the region $|J_0| < a$, $|\sigma_0| > b$, equations (1.21) and (1.22) are used.

The simultaneous solution of (1.21) and (1.23) gives $|J_0| = a$, $|\sigma_0| = b$, which with substitution of $|J_0| = a$ in the first relation of (1.24) transform it to $\langle\varepsilon_{ij}\rangle = \lambda_2(1 + c_1^*/\gamma)\langle\tilde{\sigma}_{ij}\rangle$. Considering the conditions of differentiability (1.22) and (1.24), it follows that the surfaces (1.21) and (1.23) smoothly close down to points with coordinates $|J_0| = a$, $|\sigma_0| = b$.

Equation (1.21) describes an ellipse with its center at the origin in the σ_0, J_0 plane, and (1.23) describes part of an ellipse

$$(J_0 + \hat{k}\beta^*)^2 + \alpha\sigma_0^2 = k^{*2}\beta, \tag{2.3}$$

lying in the half-plane $J_0 > 0$, and its mirror image in the half-plane $J_0 < 0$.

In Fig. 1 the solid lines AB and A'B' correspond to (1.23), and the conditions for the deformation of inclusions (2.2). Lines AA' and BB' answer to the plasticity conditions of a composite with rigid inclusions (1.21). The broken lines AB and A'B' are part of the ellipse (1.21) in the region where the porous body behavior is described by (1.23) and (1.24). We have a similar situation for the broken lines APB and A'P'B' corresponding to (1.23). It follows that for $k_1 > k_0$ and $\gamma^* > 0$, the plasticity condition for the composite is depicted by solid lines ABB'A', comprising parts of ellipses (1.21) and (1.23). The function γ^* dies out with increasing porosity. Therefore, in the decompacting process the value $c_2 = c_2^*$ can be reached, for which $\gamma^* = 0$. In this case the solid lines AB and A'B' degenerate to points on ellipses (1.21) and (1.23).

If $\gamma^* < 0$, (2.2) cannot be fulfilled on the surface (1.23). The inclusions behave as rigid particles and the deformation of the composite is described by (1.21) and (1.22) for all loading paths. In Fig. 1, the inner ellipse "breaks away" from the outer and with subsequent increase in porosity degenerates to the point where $k^* = 0$.

Now let $k_1 < k_0$, $\hat{k} > 0$. From (1.17) and the condition $I_1 = 0$ it follows that $I = I_0 = 0$. That is, the inclusions begin to deform simultaneously with the matrix. Ellipse (2.3) in the σ_0, J_0 plane shifts down from the origin along the J_0 axis (lines A'P'B' in Fig. 2). Equation (1.23) describes the portion MP'N of it in the region $J_0 > 0$ and its symmetric reflection MPN in the region $J_0 < 0$, that is, the closed curve MP'NP having two cusped points M and N with coordinates $J_0 = 0$, $|\sigma_0| = \sqrt{(k^{*2}\beta - \hat{k}\beta^*)/\alpha}$. $J_0 = 0$ in (1.24) gives rise to an indefinite value for ε_{ij}, corresponding to points M and N on surface (1.23).

The broken line 1 in Fig. 2 is the plasticity condition (1.21) for a porous material with rigid inclusions. For $k_1 = k_0$ or $c_1^* = 0$, (1.23) and (1.24) are transformed to relations for plastic deformation of a porous medium with a homogeneous solid phase [2, 7]:

$$J_0^2 + \alpha\sigma_0^2 = \beta k_0^2, \quad \langle\tilde{\varepsilon}_{ij}\rangle = \lambda\langle\tilde{\sigma}_{ij}\rangle, \quad \varepsilon_0 = \lambda 3\alpha\sigma_0. \tag{2.4}$$

The plasticity surface (2.4) is depicted by broken line 2.

We examine the process of unilateral compression of the material in a compression mould, when the transverse displacement is held equal to zero and friction is not taken into account. Under these conditions the loading equation (2.4) is used to calculate pressure as a function of porosity. It has been shown that the theoretical press curves agree satisfactorily with experiment [7, 8].

For a porous material with rigid inclusions we obtain, from (1.21) and (1.22), the stress in the direction of the deformation

$$\sigma_1 = Y_0(1 - c_2)\sqrt{\frac{2}{3c_2} + \frac{2 + 3c_1^*}{3 + 2c_2 - 3c_1^*(1 - c_2)}} \tag{2.5}$$

[Y_0 is the plastic limit of the matrix under simple tension (compression)].

The solid line in Fig. 3 is the press curve for porous titanium with 15% rigid inclusions of Cr_3C_2 [9]. The broken line was constructed according to (2.5) for $c_1^* = 0.15$, $Y_0 = 240$ MPa [10].

Comparison of the theoretical and experimental relations makes it clear that the value of the porosity for the pressure computed from (2.5) does not differ from the corresponding experimental value by more than 5%. Experimental data reflecting the peculiarities of transformation of the inclusions from a rigid state to a plastic one are not available for porous materials.

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THEORY OF ELASTIC-PLASTIC DEFORMATION OF RANDOMLY REINFORCED COMPOSITE MATERIALS

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Using the methods of the mechanics of random inhomogeneous media, we study the elastic-plastic properties of a composite material containing randomly oriented ellipsoidal inclusions. The analogous problem for composites with spherical inclusions and matrix mixtures was solved in [1].

1. Let a composite material occupying volume V and bounded by surface S be formed of an elastic-plastic matrix and randomly oriented ellipsoidal inclusions of identical form. The governing equations for the materials of both components, bonded together with ideal adhesion, are given by

$$s_{ij} = 2\mu_m(e)e_{ij},\ \sigma_{pp} = 3K_m\varepsilon_{pp},\ s_{ij} = 2\mu_f(e)e_{ij},\ \sigma_{pp} = 3K_f\varepsilon_{pp}. \tag{1.1}$$

Here $s_{ij} = \sigma_{ij} - (1/3)\delta_{ij}\sigma_{pp}$; $e_{ij} = \varepsilon_{ij} - (1/3)\delta_{ij}\varepsilon_{pp}$; σ_{ij}, ε_{ij} are the components of the stress and deformation tensors; $\mu_{m,f}(e)$ are the plastic shear moduli; $K_{m,f}$ are the bulk moduli of the material components ($K_{m,f}$ = const); $e = \sqrt{e_{ij}e_{ij}}$. The index m refers to the matrix material; f to that of the inclusions.

We will describe the structure of the composite by using the indicator function $\kappa(\mathbf{r})$, which is equal to zero in the matrix volume V_m and to unity in the inclusion volume V_f. In addition, the spatial position of the ellipsoids is given by a collection of indicator functions $\varkappa_1(\mathbf{r}), \varkappa_2(\mathbf{r}), \ldots, \varkappa_n(\mathbf{r})$. Each function $\kappa_s(\mathbf{r})$ is equal to unity in the volume V_s of all ellipsoids oriented in direction s and is equal to zero outside of this volume. Clearly $\varkappa(\mathbf{r}) = \sum_{s=1}^{n} \varkappa_s(\mathbf{r})$. By using $\kappa(\mathbf{r})$, (1.1) can be written in the form

$$\begin{aligned} s_{ij}(\mathbf{r}) &= 2(\mu_m(e) + (\mu_f(e) - \mu_m(e))\varkappa(\mathbf{r}))e_{ij}(\mathbf{r}),\\ \sigma_{pp}(\mathbf{r}) &= 3(K_m + (K_f - K_m)\varkappa(\mathbf{r}))\varepsilon_{pp}(\mathbf{r}). \end{aligned} \tag{1.2}$$

All of the indicator functions, stresses and deformations are assumed to be statistically uniform and ergodic random fields, and their expectation values are replaced by volume-averaged values [2]:

$$\langle f\rangle = \frac{1}{V}\int_V f(\mathbf{r})\,d\mathbf{r},\quad \langle f\rangle_{m,f,s} = \frac{1}{V_{m,f,s}}\int_{V_{m,f,s}} f(\mathbf{r})\,d\mathbf{r}\ (s = 1, 2, \ldots, n).$$

To find the macroscopic governing equations and the effective characteristics of these composites it is necessary to establish a connection between the macroscopic quantities $\langle\sigma_{ij}\rangle$ and $\langle\varepsilon_{ij}\rangle$:

$$\langle\sigma_{ij}\rangle = E^*_{ijkl}\langle\varepsilon_{kl}\rangle, \tag{1.3}$$

where $E_{ijk\ell}^*$ are the components of the plastic moduli tensor, a function of the numerical characteristics of the random deformation field $\varepsilon_{ij}(\mathbf{r})$. Here and below an asterisk denotes the root mean square of the quantity.

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